Edge Focusing of Orbit Bumps and Doglegs

W. Chou , January 23, 2003

- The edge of a rectangular bending magnet gives focusing in the non-deflecting plane (for orbit bumps, V-plane; for doglegs, H-plane). It is a drift in the deflecting plane.

- In the non-deflecting plane, the focusing strength is:

$$1/f = (1/\rho) \times \tan(\theta/2) \cong \theta^2 / 2L$$

  f = focal length
  ρ = bending radius
  θ = bending angle
  L = bending magnet length

- This explains why the doglegs have bigger effects on the Booster lattice than the orbit bumps:
  - The bending angle θ of doglegs is larger (dogleg: 56.55 mrad, *vs.* orbit bump: 42.7067 mrad)
  - The length L of doglegs is shorter (dogleg: 20.3 cm, *vs.* orbit bump: 46.443 cm)
  - So the edge focusing strength $1/f$ of a dogleg is about a factor of 4 larger than that of an orbit bump (dogleg: 0.008 m^{-1}, *vs.* orbit bump: 0.002 m^{-1})
  - Furthermore, there are 8 doglegs, versus only 4 orbit bumps (see the additive feature discussed below)

- This also explains why the dogleg effects vanish quickly as the beam is accelerated: The doglegs are DC magnets. The bending angle θ is inversely proportional to the beam energy.

So the focusing strength $1/f$ is inversely proportional to the square of the beam energy.

- The beta-wave at location 2 caused by a thin quadrupole at location 1 is: (to the first order)

$$\Delta\beta = -q \times \beta_1 \times \beta_2 \times \sin(2\psi)$$

q = thin quadrupole strength = $B'L/B\rho = 1/f$

β_1, β_2 = beta-functions of the unperturbed lattice

ψ = phase advance from 1 to 2

- Calculation for one Booster orbit bump:
  Turn on orbit bump No. 1, watch downstream element 140:

$$\theta = 0.0427067 \text{ rad}$$
$$L = 0.46443 \text{ m}$$
$$1/f = 0.002 \text{ m}^{-1}$$
$$\beta_1 = 20 \text{ m}$$
$$\beta_2 = 20 \text{ m}$$
$$2\psi = 7\pi/2$$
$$\Delta\beta = 0.8 \text{ m}$$

  Compared with the MAD output:

$$\Delta\beta = 0.849 \text{ m}$$

- A surprise (at least to me):
  Either positive bending or negative bending, the edge is ALWAYS FOCUSING.

- So the edge focusing of the 8 doglegs will add up in the H-plane. That of the 4 orbit bumps will add up in the V-plane. Both lead to a large beta-wave. The former also leads to a large dispersion wave, as pointed out by Sasha.